Costly Financial Intermediation and the Interest Rate Spread

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September 28, 2025

Problem Setup

Consider a two-period economy with two types of households:

$$\begin{cases} \text{Type A (fraction } \theta) : y_0^A \gg y_1^A \quad \text{(rich today, poor tomorrow)} \\ \text{Type B (fraction } 1-\theta) : y_1^B \gg y_0^B \quad \text{(poor today, rich tomorrow)} \end{cases}$$

Both maximize discounted log-utility:

$$U(C_0^i, C_1^i) = \log(C_0^i) + \beta \log(C_1^i), \quad i \in \{A, B\}.$$

For some further mathematical intuition

$$\max_{C_0^i, C_1^i} \log(C_0^i) + \beta \log(C_1^i) \tag{1}$$

$$\text{s.t} \tag{2}$$

$$C_0^i + \frac{1}{1+r_d} C_1^i = y_0^i + \frac{1}{1+r_d} y_1^i \tag{3}$$

Type A households naturally want to *save* (lend), while Type B want to *borrow*. If financial markets were frictionless, a single equilibrium interest rate would perfectly match the supply of savings with the demand for borrowing.

The Reality: A Positive Spread

In practice, financial intermediaries impose a wedge between borrowing and lending rates:

$$r_l > r_d, \qquad (1 + r_l) = (1 + r_d)(1 + x), \qquad x \approx r_l - r_d.$$

The spread x captures the *inefficiency of intermediation*.
- Savers (Type A) earn the low deposit rate r_d.
- Borrowers (Type B) pay the high loan rate r_l.

Individual Optimization

Type A (savers). Optimal consumption solves:

$$C_0^A = \frac{y_0^A + \frac{1}{1+r_d} y_1^A}{1+\beta}, \qquad C_1^A = \beta(1+r_d)C_0^A.$$

Type A saves positively: $S_0^A = y_0^A - C_0^A > 0$.

Type B (borrowers). Optimal consumption solves:

$$C_0^B = \frac{y_0^B + \frac{1}{1+r_l} y_1^B}{1+\beta}, \qquad C_1^B = \beta(1+r_l)C_0^B.$$

Type B borrows at $t = 0$: $S_0^B = y_0^B - C_0^B < 0$.

Market-Clearing and Equilibrium Rates

Deriving bond market clearing delivers equilibrium deposit and loan rates:

$$\mathcal{D}^{bond} = \int_0^\theta S_0^A di = \theta \left(y_0^A - \frac{y_0^A + \frac{1}{1+r_d} y_1^A}{1+\beta} \right)$$

$$\mathcal{S}^{bond} = \int_\theta^1 S_0^A di = (1-\theta) \left(y_0^B - \frac{y_0^B + \frac{1}{1+r_l} y_1^B}{1+\beta} \right)$$

Letting $\mathcal{S}^{bond} = \mathcal{D}^{bond}$ and solving for r_l and r_d, we obtain the optimal rates of lending and borrowing, which I have annotated with a $*$ superscript.

$$r_d^* = \frac{\theta y_1^A + \frac{1-\theta}{1+x} y_1^B}{\beta(\theta y_0^A + (1-\theta) y_0^B)} - 1,$$

$$r_l^* = \frac{(1+x)\theta y_1^A + (1-\theta) y_1^B}{\beta(\theta y_0^A + (1-\theta) y_0^B)} - 1.$$

In the "ideal" case ($x = 0$), these rates coincide. In reality ($x > 0$), they diverge.

Aggregate Utility Comparison between a Zero Interest Rate Gap Economy and an Interest Rate Gap Economy

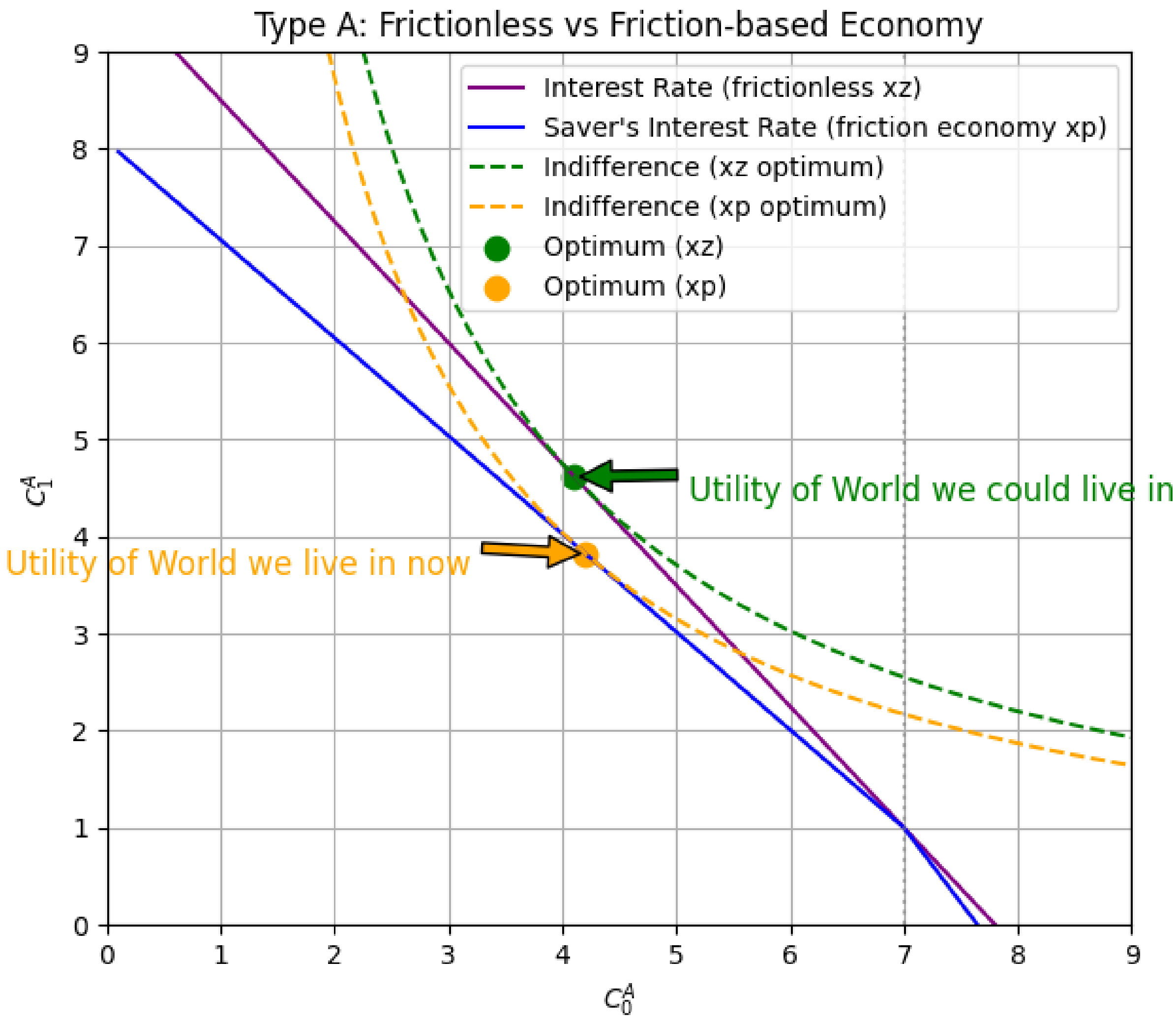
The utility of the economy at large can be written as

$$\begin{aligned} U_{aggregate} &= \int_0^1 U(C_0^i, C_1^i) di && (4) \\ &= \theta \left\{ \log(C_0^A) + \beta \left(\log(C_1^A) \right) \right\} + (1-\theta) \left\{ \log(C_0^B) + \beta \left(\log(C_1^B) \right) \right\} && (5) \end{aligned}$$

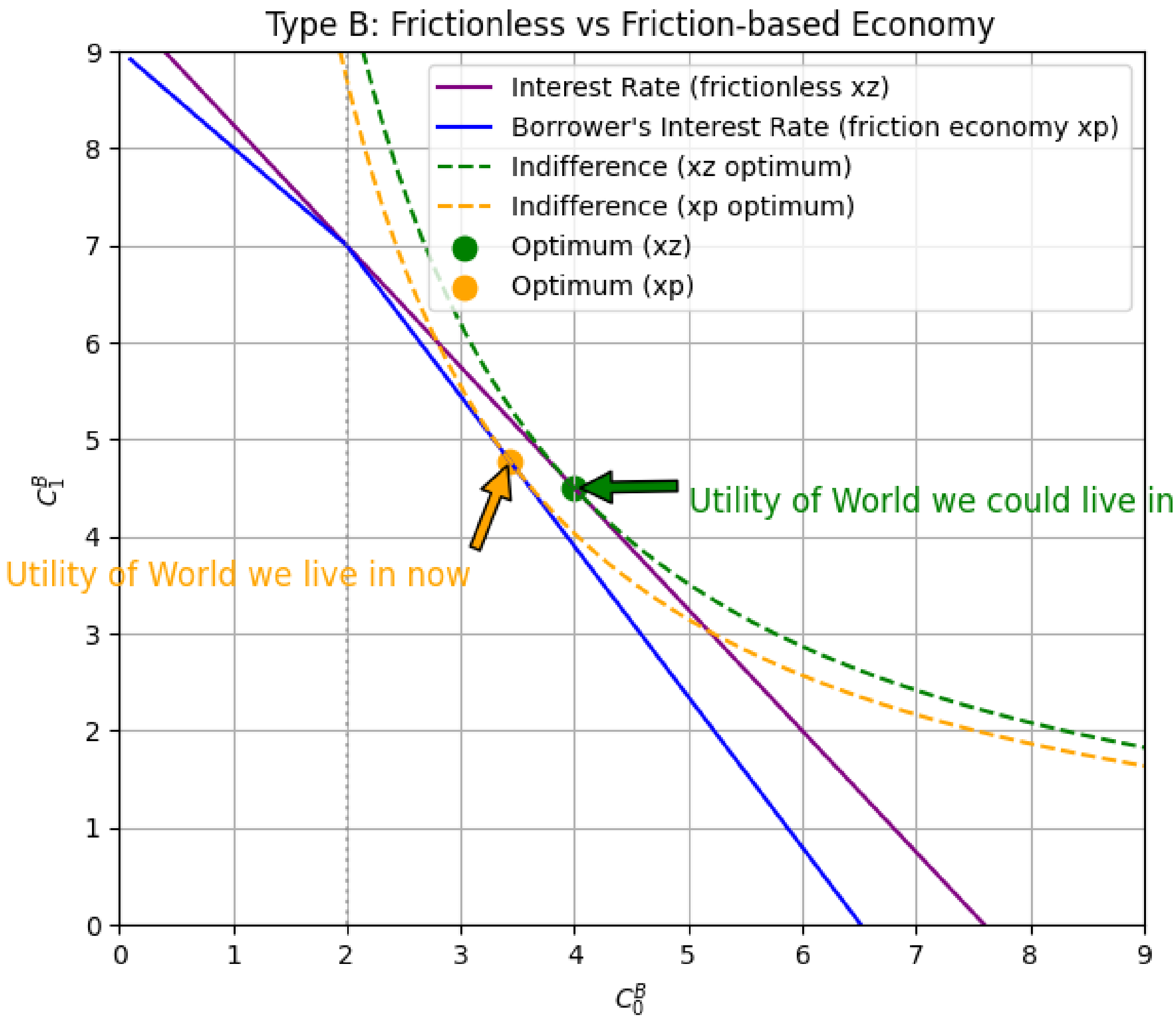
I will denote xp as having a positive interest rate difference (i.e. $r_l - r_d > 0$) and xz as a zero interest rate gap (i.e $r_l - r_d = 0$). Since we know that $r^*_{d,xp} < r_{xp} < r^*_{l,xp}$, we can plug these back in to our optimal conditions and we compare the values of optimal utility.

$$U_{A,xp} < U_{A,xz} \quad U_{B,xp} < U_{B,xz}$$

Graphical Interpretation



The blue and purple lines represent budget constraints as defined in the problem setup. The blue line is a piecewise function that has a slope of $\frac{1}{1+r_d}$ up until the point of $(y_0, y_1) = (7, 1)$. Beyond that point, the slope changes to $\frac{1}{1+r_l}$. The green and orange lines are our indifference curves which represent utility.



Similar story here. The blue and purple lines represent budget constraints as defined in the problem setup. The blue line is a piecewise function that has a slope of $\frac{1}{1+r_d}$ up until the point of $(y_0, y_1) = (2, 7)$. Beyond that point, the slope changes to $\frac{1}{1+r_l}$. The green and orange lines are our indifference curves which represent utility.

Economic Interpretation

In the ideal frictionless economy, the interest rate is unique, and savings and borrowing are efficiently allocated. In the real economy, intermediaries create a spread to pay for tellers, branches, credit analysts, etc. Savers earn less, borrowers pay more, and society as a whole is worse off.

Moral: We don't live in the "ideal world" where credit flows at a single fair rate. Instead, intermediation costs show up as a wedge between r_d and r_l, lowering utility for everyone who is borrowing or lending money.